

NEURAL EXCITABILITY, SPIKING AND BURSTING

EUGENE M. IZHIKEVICH

<https://doi.org/10.1142/S0218127400000840> | Cited by: 1361

Next 

 PDF/EPUB

 Tools  Share  Recommend To Library

神经兴奋性、峰值和簇发放的分岔机制

Section 5: 其他类型

NEURAL EXCITABILITY, SPIKING AND BURSTING

Author: Eugene M. Izhikevich

DOI: 10.1142/S0218127400000840

Dates: Received June 9, 1999

Translated by Na Zhao, School of Mathematics, SCUT.

Link to the Journal: [10.1142/S0218127400000840](https://doi.org/10.1142/S0218127400000840)

International Journal of Bifurcation and Chaos

Vol. 24, No. 8 (2014) 1440003

2023 年 3 月 28 日

* 这篇论文发表于 1999 年，引用量已经高达两千多。Izhikevich 在该文里详细介绍了神经元兴奋性、峰值和簇发放所涉及的详细分岔机制。对于神经动力学的读者而言，该文提供了详细的理论基础。由于该文内容冗长，特意将其拆封成多个部分，以便读者准确定位到自己所需。这是 Sec. 5: 其他类型。

NEURAL EXCITABILITY, SPIKING AND BURSTING

EUGENE M. IZHIKEVICH

The Neurosciences Institute, 10640 John Jay Hopkins Drive, San Diego, CA 92121, USA
*Center for Systems Science Engineering, Arizona State University, Tempe, AZ 85287-7606, USA**

本文综述了神经元产生动作电位 (尖峰) 所涉及的分岔机制。我们展示了分岔的类型如何决定细胞的神经计算特性。例如, 当稳态接近鞍-结点分岔时, 细胞可以以任意低频发放全有或全无尖峰, 它具有明确定义的阈值流形, 并且充当积分器; 即输入脉冲的频率越高, 它放电的越快。相反, 当稳态接近 Andronov-Hopf 分岔时, 细胞在特定频率范围内发放, 其尖峰不是全有或无, 它没有明确定义的阈值流形, 它可以响应抑制脉冲充当谐振器; 即它优先响应输入的某个 (共振) 频率。增加输入频率实际上可能会延迟或终止其触发。

我们还描述了神经簇发放现象, 使用几何分岔理论扩展了现有的簇发放分类, 包括许多新类型。我们讨论了 burster 的类型如何定义其神经计算属性, 并且我们展示了不同的 burster 可以不同地交互、同步和处理信息。

Contents

1. 周期性簇发放: 刺猬极限环	2
1.1. 蓝天灾难	2
1.2. 鞍焦点同宿轨道	2
2. 准周期性的簇发放	2
3. 混沌的簇发放	2

* Eugene.Izhikevich@nsi.edu;

在这一节中，我们简要地提到不能用快慢系统 (21) 描述的簇发放事件。我们区分三种情况：周期性的、准周期性的和混沌的簇发放行为。

1. 周期性簇发放：刺猬极限环

当一个系统具有图 113 中描述的刺猬极限环吸引子时，往往会出现周期性的簇发放。我们把这样的簇发放器称为刺猬簇发放器。上一节中考虑的许多快慢速簇发放器在 μ 小的时候与刺猬吸引子有周期性的动态关系，而当 μ 中等或大的时候，它们作为刺猬簇发放器持续存在。

即使是二维系统，例如那些具有扭曲的快速零斜线的系统，也可以表现出刺猬簇发放；见图 114。以下是两个刺猬案例。

1.1. 蓝天灾难

当神经系统接近蓝天大灾难时，也会出现刺猬吸引子 (图 30)，在相空间的尖峰区域有折叠极限环分岔。事实上，矢量场是沿着消失的周期指向的，因此解花了大部分时间围绕它旋转，从而发放了尖峰；见图 115。该系统表现出周期性的簇发放行为，尽管它不能被分解为形式为 (21) 的快速和慢速子系统。因此，这种簇发放不属于快速慢速类型。

1.2. 鞍焦点同宿轨道

现在考虑一个靠近鞍焦点同宿轨道分岔的神经系统；见图 30 的顶部。在 Shilnikov 发现的某些相当普遍的条件下 (见 [Kuznetsov, 1995])，鞍焦点有次级同宿轨道，可以 (通过次级鞍焦点同宿分岔) 为对应于两次、三次等的稳定环，见图 116。

这是 FitzHugh-Nagumo 偏微分方程 [Hastings, 1976; Evans et al., 1982; Feroe, 1982] 中产生复杂尖峰的机制，它可能与龙虾随机胃神经节的 LP 细胞的簇发放机制有关 [Guckenheimer et al., 1997]。

2. 准周期性的簇发放

如果有一个连续函数 $q(\theta_1, \dots, \theta_k)$ ，在每个参数中都是 2π 周期性的，那么一个连续的节奏信号 $x(t)$ 就是准周期性的，从而

$$x(t) = q(\omega_1 t, \dots, \omega_k t), \quad \forall t \geq 0,$$

其中 $\Omega = (\omega_1 t, \dots, \omega_k t)^T \in \mathbb{R}^k$ 是一个频率向量 [Samoilenko, 1991; Izhikevich, 1999a]。准周期活动对应于一个环面吸引子，它通常看起来像簇发放波或主轴波；见图 117。它的形状取决于函数 q 和频率向量 Ω 。

环面吸引子可能会出现，例如通过超临界的 Neimark-Sacker 分岔。它也可以通过具有同宿环面结构的折叠极限环分岔出现 (图 30)。当周期消失时，系统表现出类似于图 115 中描述的“蓝天”簇发放的行为。图 118 中描述了一些额外的情况。

3. 混沌的簇发放

许多混沌的信号类似于环-环的簇发放；见图 119。点-环簇发放发生，例如在 Pomeau-Manneville 间歇性的存在下。许多靠近鞍焦点同宿轨道分岔的系统也表现出点-环混沌簇发放 [Kuznetsov, 1995]，具有类似 Rössler 的吸引子；见图 120。另一方面，许多快慢速簇发放器具有混沌的动力学特性。例如，“折叠/同宿”簇发放器可能具有鞍结构 [Terman, 1991, 1992]，因此表现为混沌的簇发放。Terman 的论点可以被推广到大多数“*/同宿”簇发放器。



图 113. 一个刺猬状极限环吸引子通常对应于簇发放行为 (来自 [Hoppensteadt & Izhikevich, 1997])。

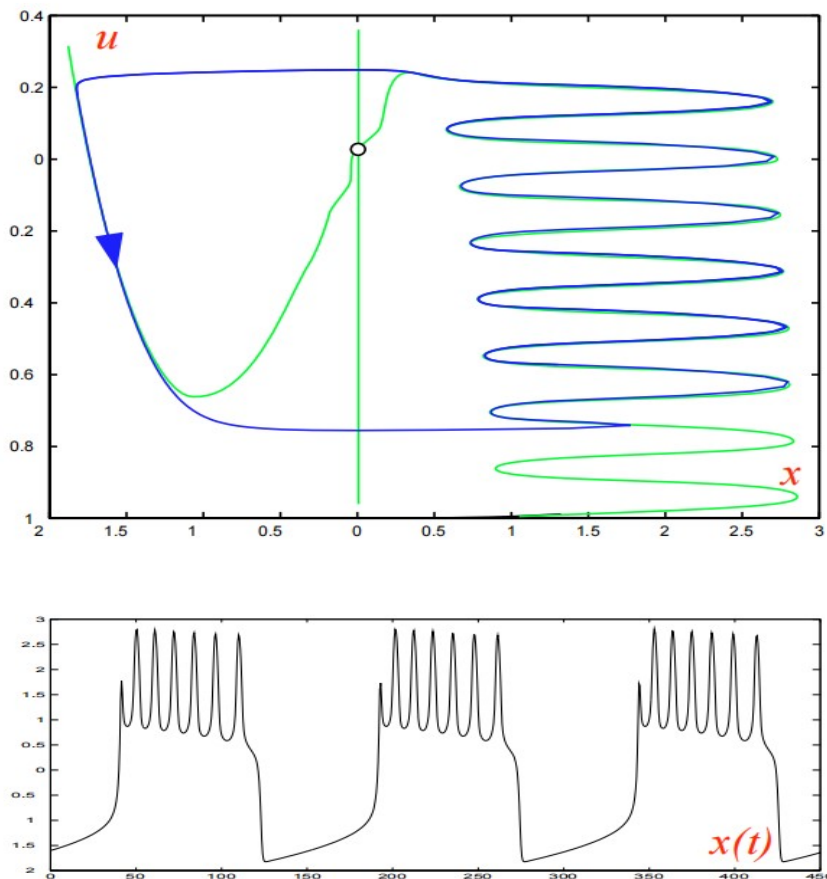


图 114. 平面松弛系统 $\dot{x} = x - x^3/3 - u + 4S(x)\cos 40u$, $\dot{u} = \mu x$, 其中 $S(x) = 1/(1 + e^{5(1-x)})$, $\mu = 0.01$ 中的快和慢零斜 (绿色) 和刺猬极限环吸引子 (蓝色)。

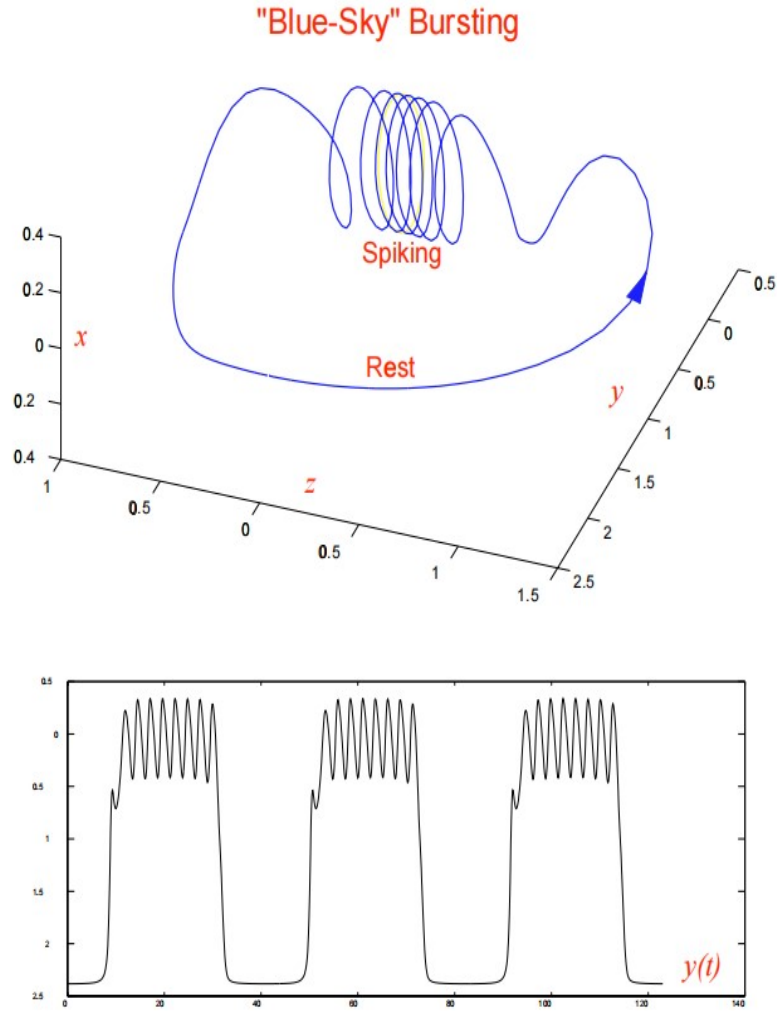


图 115. 蓝天型的刺猬簇发放。Gavrilov-Shilnikov 模型的模拟 [Kuznetsov, 1995, p. 256]。参数： $b = 10, \varepsilon = 0.02, \mu = 0.5$ 。

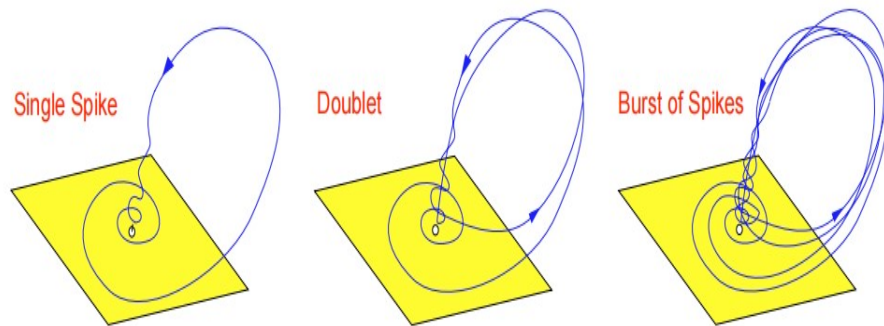


图 116. 次级同宿轨道到鞍焦点平衡可以分岔为对应于两次、三次 (未显示) 或尖峰簇发放的周期性解。

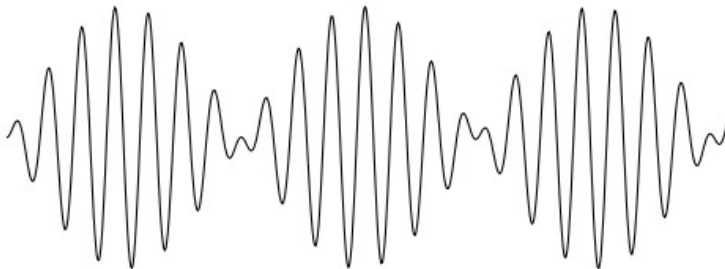


图 117. 一个准周期信号的例子。 $x(t) = \sin\omega_1 t \sin\omega_2 t$, 频率矢量 $(\omega_1, \omega_2) = (1, \sqrt{2}/20)$ 。

"Homoclinic Torus" Bursting

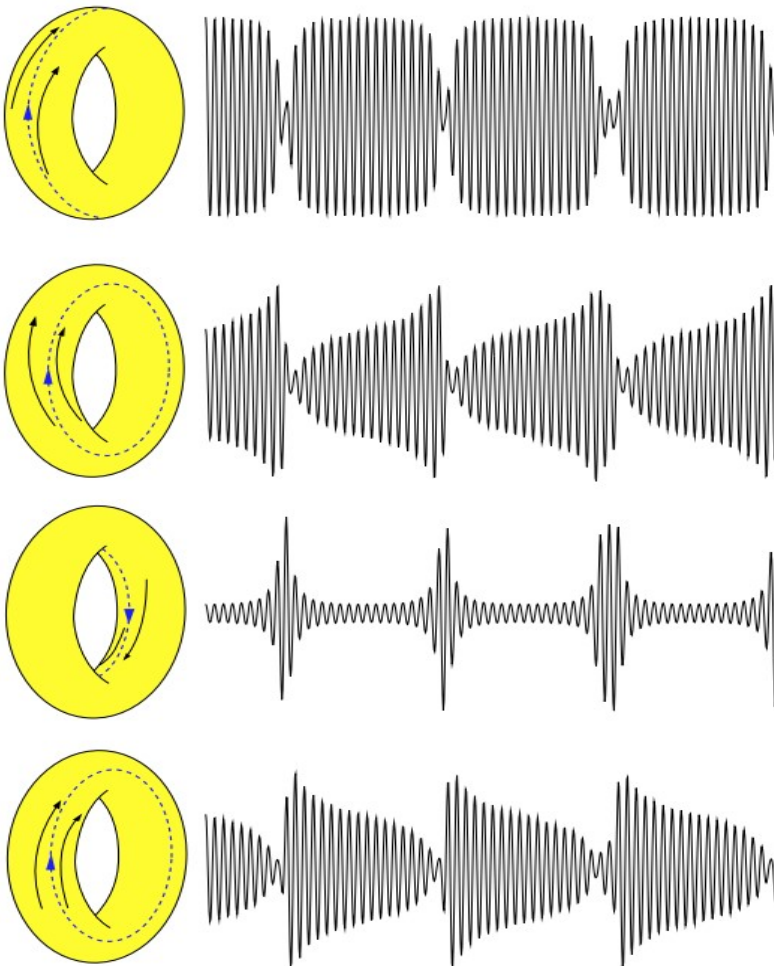


图 118. "同宿环面" 簇发放。该模式取决于消失的折叠极限环的位置 (蓝色虚线圆)。

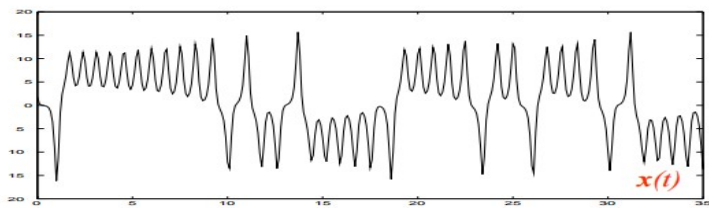


图 119. 洛伦兹系统的解类似于混沌的周期簇发放。

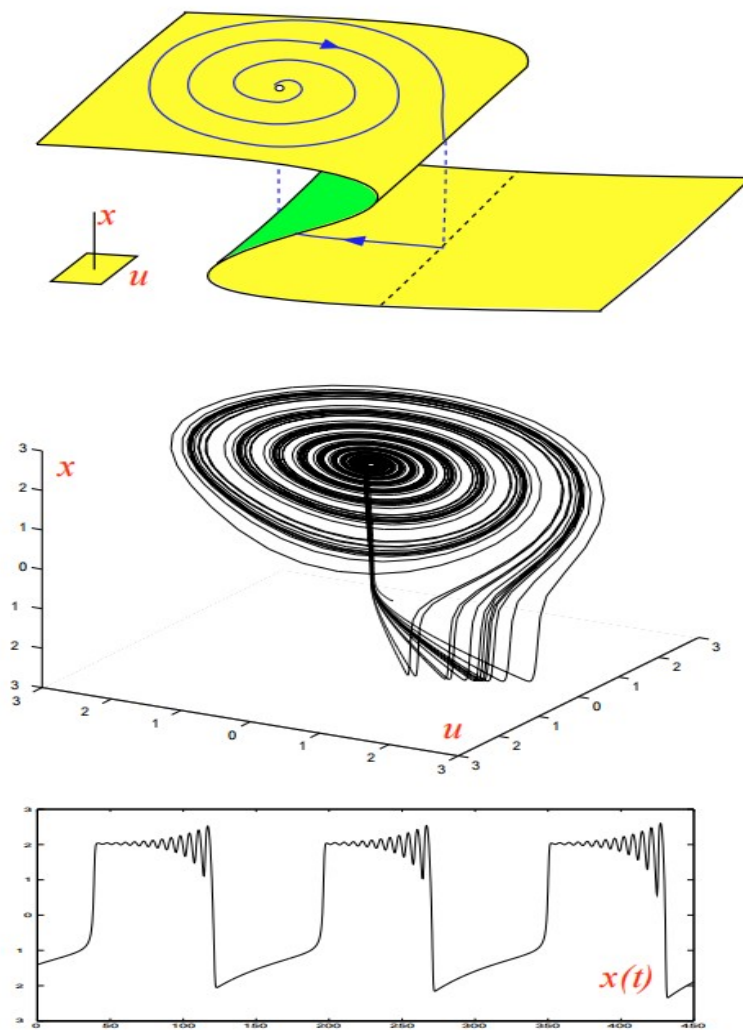


图 120. 混沌的簇发放在鞍焦点同宿轨道分岔附近。系统模拟

$$\dot{x} = x - x^3/3 + I - Reu, \dot{u} = \mu(S(x)(1 + \omega i)u - u/2), \text{ 其中 } u \in \mathbb{C}, S(x) = 1/(1 + e^{-5x}), \omega = -10, I=0.75, \mu = 0.1$$

参考文献

- Abarbanel, H. D. I., Huerta, R., Rabinovich, M. I., Rulkov, N. F., Rowat, P. F. & Selverston, A. I. [1996] "Synchronized action of synaptically coupled chaotic model neurons," *Neural Comput.* 8, 1567-1602.
- Alexander, J. C. & Cai, D. [1991] "On the dynamics of bursting systems," *J. Math. Biol.* 29, 405-423.
- Alexander, J. C., Doedel, E. J. & Othmer, H. G. [1990] "On the resonance structure in a forced excitable system," *SIAM J. Appl. Math.* 50, 1373 - 1418.
- Arnold, V. I. [1982] *Geometrical Methods in the Theory of Ordinary Differential Equations* (Springer-Verlag, NY); Russian original [1977] *Additional Chapters of the Theory of Ordinary Differential Equations*, Moscow.
- Arnold, V. I., Afrajmovich, V. S., Il' yashenko, Yu. S. & Shil'nikov, L. P. [1994] "Bifurcation theory," in *Dynamical Systems V. Bifurcation Theory and Catastrophe Theory*, ed. Arnold, V. I. (Springer-Verlag, NY).
- Aronson, D. G., Ermentrout, G. B. & Kopell, N. [1990] "Amplitude response of coupled oscillators," *Physica D* 41, 403-449.
- Baer, S. M., Erneux, T. & Rinzel, J. [1989] "The slow passage through a Hopf bifurcation: Delay, memory effects, and resonances," *SIAM J. Appl. Math.* 49, 55-71.
- Baer, S. M., Rinzel, J. & Carrillo, H. [1995] "Analysis of an autonomous phase model for neuronal parabolic bursting," *J. Math. Biol.* 33, 309-333.
- Bedrov, Y. A., Akoev, G. N. & Dick, O. E. [1992] "Partition of the Hodgkin-Huxley type model parameter space into regions of qualitatively different solutions," *Biol. Cybern.* 66, 413-418.
- Belair, J. & Holmes, P. [1984] "On linearly coupled relaxation oscillations," *Quarterly of Appl. Math.* 42, 193-219.
- Bertram, R. [1993] "A computational study of the effects of serotonin on a molluscan burster neuron," *Biol. Cybern.* 69, 257-267.
- Bertram, R., Butte, M. J., Kiemel, T. & Sherman, A. [1995] "Topological and phenomenological classification of bursting oscillations," *Bull. Math. Biol.* 57, 413-439.
- Booth, V., Carr, T. W. & Erneux, T. [1997] "Nearthreshold bursting is delayed by a slow passage near a limit point," *SIAM J. Appl. Math.* 57, 1406-1420.
- Butera Jr., R. J., Clark Jr., J. W. & Byrne, J. H. [1996] "Dissection and reduction of a modeled bursting neuron," *J. Comput. Neurosci.* 3, 199-223.
- Butera Jr., R. J., Clark Jr., J. W. & Byrne, J. H. [1997] "Transient responses of a modeled bursting neuron: Analysis with equilibrium and averaged nullclines," *Biol. Cybern.* 77, 307-322.
- Canavier, C. C., Clark, J. W. & Byrne, J. H. [1991] "Simulation of the bursting activity of neuron-R15 in aplysia —role of ionic currents, calcium balance, and modulatory transmitters," *J. Neurophysiol.* 66, 2107-2124.
- Carpenter, G. A. [1979] "Bursting phenomena in excitable membranes," *SIAM J. Appl. Math.* 36, 334-372.
- Chay, T. R. & Keizer, J. [1983] "Minimal model for membrane oscillations in the pancreatic β -cell," *Biophys. J.* 42, 181-190.
- Connor, J. A. & Stevens, C. F. [1971] "Prediction of repetitive firing behavior from voltage-clamped data on an isolated neurone soma," *J. Physiol. Lond.* 214, 31-53.

- Del Negro, C. A., Hsiao, C.-F., Chandler, S. H. & Garfinkel, A. [1998] "Evidence for novel bursting mechanism in rodent trigeminal neurons," *Biophys. J.* 75, 174-182.
- de Vries, G. [1998] "Multiple bifurcations in a polynomial model of bursting oscillations," *J. Nonlin. Sci.* 8, 281-316.
- Ermentrout, G. B. [1996] "Type I membranes, phase resetting curves, and synchrony," *Neural Comput.* 8, 979-1001.
- Ermentrout, G. B. [1998] "Linearization of F-I curves by adaptation," *Neural Comput.* 10, 1721 - 1729.
- Ermentrout, G. B. & Kopell, N. [1986a] "Parabolic bursting in an excitable system coupled with a slow oscillation," *SIAM J. Appl. Math.* 46, 233-253.
- Ermentrout, G. B. & Kopell, N. [1986b] "Subcellular oscillations and bursting," *Math. Biosci.* 78, 265-291.
- Evans, J., Fenichel, N. & Feroe, J. [1982] "Double impulse solutions in nerve axon equations," *SIAM J. Appl. Math.* 42, 219-234.
- Fenichel, N. [1971] "Persistence and smoothness of invariant manifolds for flows," *Ind. Univ. Math. J.* 21, 193-225.
- Feroe, J. A. [1982] "Existence and stability of multiple impulse solutions of a nerve equation," *SIAM J. Appl. Math.* 42, 235-246.
- FitzHugh, R. [1955] "Mathematical models of threshold phenomena in the nerve membrane," *Bull. Math. Biophys.* 17, 257-278.
- Frankel, P. & Kiemel, T. [1993] "Relative phase behavior of two slowly coupled oscillators," *SIAM J. Appl. Math.* 53, 1436-1446.
- Grasman, J. [1987] *Asymptotic Methods for Relaxation Oscillations and Applications* (Springer-Verlag, NY).
- Guckenheimer, J., Harris-Warrick, R., Peck, J. & Willms, A. [1997] "Bifurcations, bursting and spike frequency adaptation," *J. Comput. Neurosci.* 4, 257-277.
- Gutfreund, Y., Yarom, Y. & Segev, I. [1995] "Subthreshold oscillations and resonant frequency in guinea-pig cortical neurons: Physiology and modeling," *J. Physiol. London* 483, 621-640.
- Gutkin, B. S. & Ermentrout, G. B. [1998] "Dynamics of membrane excitability determine interspike interval variability: A link between spike generation mechanisms and cortical spike train statistics," *Neural Comput.* 10, 1047-1065.
- Hansel, D., Mato, G. & Meunier, C. [1995] "Synchrony in excitatory neural networks," *Neural Comput.* 7, 307-335.
- Hassard, B. D. [1978] "Bifurcation of periodic solutions of the Hodgkin - Huxley model for the squid giant axon," *J. Theoret. Biol.* 71, 401-420.
- Hassard, B. D., Kazarinoff, N. D. & Wan, Y. H. [1981] *Theory and Applications of Hopf Bifurcation* (Cambridge University Press, Cambridge).
- Hastings, S. [1976] "On the existence of homoclinic and periodic orbits for FitzHugh-Nagumo equations," *Quart. J. Math. (Oxford)* 27, 123-134.
- Hindmarsh, J. L. & Rose, R. M. [1984] "A model of neuronal bursting using three coupled first order differential equations," *Philos. Trans. R. Soc. London, Ser. B* 221 87-102.
- Hodgkin, A. L. [1948] "The local electric changes associated with repetitive action in a non-medulated axon," *J. Physiol.* 107, 165-181.
- Hodgkin, A. L. & Huxley, A. F. [1952] "A quantitative description of membrane current and application to conduction and excitation in nerve," *J. Physiol.*

117, 500–544.

Holden, L. & Erneux, T. [1993a] "Slow passage through a Hopf bifurcation: From oscillatory to steady state solutions," *SIAM J. Appl. Math.* 53, 1045–1058.

Holden, L. & Erneux, T. [1993b] "Understanding bursting oscillations as periodic slow passages through bifurcation and limit points," *J. Math. Biol.* 31, 351–365.

Holden, A. V., Hyde, J. & Muhamad, M. [1991] "Equilibria. Periodicity, bursting and chaos in neural activity," *Proc. 9th Summer Workshop on Mathematical Physics*, Vol. 1, pp. 96–128.

Hoppensteadt, F. C. [1997] *An Introduction to the Mathematics of Neurons. Modeling in the Frequency Domain* (Cambridge University Press).

Hoppensteadt, F. C. [1993] *Analysis and Simulations of Chaotic Systems* (Springer-Verlag, NY).

Hoppensteadt, F. C. & Izhikevich, E. M. [1996] "Synaptic organizations and dynamical properties of weakly connected neural oscillators: I. Analysis of canonical model," *Biol. Cybern.* 75, 117–127.

Hoppensteadt, F. C. & Izhikevich, E. M. [1997] *Weakly Connected Neural Networks* (Springer-Verlag, NY).

Hoppensteadt, F. C. & Izhikevich, E. M. [1998] "Thalamo-Cortical interactions modeled by weakly connected oscillators: Could brain use FM radio principles?" *BioSyst.* 48, 85–94.

Hutcheon, B., Miura, R. M. & Puil, E. [1996] "Models of subthreshold membrane resonance in neocortical neurons," *J. Neurophysiol.* 76, 698–714.

Hutcheon, B., Miura, R. M., Yarom, Y. & Puil, E. [1994] "Low-threshold calcium current and resonance in thalamic neurons: A model of frequency preference," *J. Neurophysiol.* 71, 583–594.

И' iashenko, Iu. S. & Li, W. [1999] Nonlocal

Bifurcations Mathematical Surveys and Monographs (American Mathematical Society), Vol. 66.

Izhikevich, E. M. [2001] "Resonate-and-fire neurons," *Neural Networks*, submitted.

Izhikevich, E. M. [2000a] "Subcritical elliptic bursting of Bautin type," *SIAM J. Appl. Math.* 60, 503–535.

Izhikevich, E. M. [2000b] "Phase equations for relaxation oscillators," *SIAM J. Appl. Math.*, in press.

Izhikevich, E. M. [1999a] "Weakly connected quasiperiodic oscillators, FM interactions, and multiplexing in the brain," *SIAM J. Appl. Math.* 59, 2193–2223.

Izhikevich, E. M. [1999b] "Class 1 neural excitability, conventional synapses, weakly connected networks, and mathematical foundations of pulse-coupled models," *IEEE Trans. Neural Networks* 10, 499–507.

Izhikevich, E. M. [1999c] "Weakly pulse-coupled oscillators, FM interactions, synchronization, and oscillatory associative memory," *IEEE Trans. Neural Networks* 10, 508–526.

Izhikevich, E. M. [1998] "Supercritical elliptic bursting, slow passage effect, and assistance of noise," preprint.

Jansen, H. & Karnup, S. [1994] "A spectral analysis of the integration of artificial synaptic potentials in mammalian central neurons," *Brain Res.* 666, 9–20. Johnston, D. & Wu, S. M. [1995] *Foundations of Cellular Neurophysiology* (The MIT Press).

Kopell, N. [1995] "Chains of coupled oscillators," in *Brain Theory and Neural Networks*, ed. Arbib, M. A. (The MIT press, Cambridge, MA).

Kopell, N. & Somers, D. [1995] "Anti-phase solutions in relaxation oscillators coupled through excitatory interactions," *J. Math. Biol.* 33, 261–280.

Kowalski, J. M., Albert, G. L., Rhoades, B. K. &

Gross, G. W. [1992] "Neuronal networks with spontaneous, correlated bursting activity: Theory and simulations," *Neural Networks* 5, 805-822.

Kuznetsov, Yu. [1995] *Elements of Applied Bifurcation Theory* 2nd edition (Springer-Verlag, NY).

Levi, M., Hoppensteadt, F. C. & Miranker, W. L. [1978] "Dynamics of the Josephson junction," *Quart. J. Appl. Math.* July, 167-190.

Llinas, R. R. [1988] "The intrinsic electrophysiological properties of mammalian neurons: Insights into central nervous system function," *Science* 242, 1654-1664.

Llinas, R. R., Grace, A. A. & Yarom, Y. [1991] "In vitro neurons in mammalian cortical layer 4 exhibit intrinsic oscillatory activity in the 10- to 50-Hz frequency range," *Proc. Natl. Acad. Sci. USA* 88, 897-901.

Mishchenko, E. F., Kolesov, Yu. S., Kolesov, A. Yu. & Rozov, N. K. [1994] *Asymptotic Methods in Singularly Perturbed Systems* (Plenum Press, NY).

Morris, C. & Lecar, H. [1981] "Voltage oscillations in the Barnacle giant muscle fiber," *Biophys. J.* 35, 193-213.

Nejshtadt, A. [1985] "Asymptotic investigation of the loss of stability by an equilibrium as a pair of eigenvalues slowly cross the imaginary axis," *Usp. Mat. Nauk* 40, 190-191.

Pernarowski, M. [1994] "Fast subsystem bifurcations in a slowly varied Liénard system exhibiting bursting," *SIAM J. Appl. Math.* 54, 814-832.

Pernarowski, M., Miura, R. M. & Kevorkian, J. [1992] "Perturbation techniques for models of bursting electrical activity in pancreatic β -cells," *SIAM J. Appl. Math.* 52, 1627-1650.

Plant, R. E. [1981] "Bifurcation and resonance in a model for bursting nerve cells," *J. Math. Biol.* 11,

15-32.

Puil, E., Meiri, H., Yarom, Y. [1994] "Resonant behavior and frequency preference of thalamic neurons," *J. Neurophysiol.* 71, 575-582.

Rinzel, J. [1987] "A formal classification of bursting mechanisms in excitable systems," *Mathematical Topics in Population Biology, Morphogenesis, and Neurosciences*, eds. Teramoto, E. & Yamaguti, M., Vol. 71 of *Lecture Notes in Biomathematics* (Springer-Verlag, Berlin).

Rinzel, J. & Ermentrout, G. B. [1989] "Analysis of neural excitability and oscillations," eds. Koch, C. & Segev, I. *Methods in Neuronal Modeling* (The MIT Press, Cambridge).

Rinzel, J. & Lee, Y. S. [1986] "On different mechanisms for membrane potential bursting," *Nonlinear Oscillations in Biology and Chemistry*, ed. Othmer, H. G., *Lecture Notes in Biomathematics* (Springer-Verlag).

Rinzel, J. & Lee, Y. S. [1987] "Dissection of a model for neuronal parabolic bursting," *J. Math. Biol.* 25, 653-675.

Rinzel, J. & Miller, R. N. [1980] "Numerical calculation of stable and unstable periodic solution to the Hodgkin-Huxley equations," *Math. Biosci.* 49, 27-59.

Rush, M. E. & Rinzel, J. [1995] "The potassium ACurrent, low firing rates and rebound excitation in Hodgkin-Huxley models," *Bull. Math. Biol.* 57, 899-929.

Rush, M. E. & Rinzel, J. [1994] "Analysis of bursting in a thalamic neuron model," *Biol. Cybern.* 71, 281-291.

Samoilenko, A. M. [1991] "Elements of the mathematical theory of multi-frequency oscillations," *Mathematics and Its Applications (Soviet Series)*, Vol. 71 (Kluwer Academic, Dordrecht).

- Schechter, S. [1987] "The saddle-node separatrix-loop bifurcation," *SIAM J. Math. Anal.* 18, 1142–1156.
- Sharp, A. A., O'neil, M. B., Abbott, L. F. & Marder, E. [1993] "Dynamic clamp: Computer-generated conductances in real neurons," *J. Neurophysiol.* 69, 992–995.
- Shepherd, G. M. [1981] "Introduction: The nerve impulse and the nature of nervous function," *Neurons Without Impulses*, eds. Roberts & Bush (Cambridge University Press).
- Shepherd, G. M. [1983] *Neurobiology* (Oxford University Press, NY).
- Shorten, P. R. & Wall, D. J. N. [2000] "A Hodgkin - Huxley model exhibiting bursting oscillations," *Bull.Math. Biol.*, accepted.
- Sivan, E., Segel, L. & Parnas, H. [1995] "Modulated excitability: A new way to obtain bursting neurons," *Biol. Cybern.* 72, 455–461.
- Smolen, P., Terman, D. & Rinzel, J. [1993] "Properties of a bursting model with two slow inhibitory variables," *SIAM J. Appl. Math.* 53, 861–892.
- Softky, W. R. & Koch, C. [1993] "The highly irregular firing of cortical-cells is inconsistent with temporal integration of random EPSPs," *J. Neurosci.* 13, 334–350.
- Somers, D. & Kopell, N. [1993] "Rapid synchronization through fast threshold modulation," *Biol. Cybern.* 68, 393–407.
- Somers, D. & Kopell, N. [1995] "Waves and synchrony in networks of oscillators or relaxation and nonrelaxation type," *Physica D89*, 169–183.
- Soto-Trevino, C., Kopell, N. & Watson, D. [1996] "Parabolic bursting revisited," *J. Math. Biol.* 35, 114–128.
- Storti, D. W. & Rand, R. H. [1986] "Dynamics of two strongly coupled relaxation oscillators," *SIAM J. Appl. Math.* 46, 56–67.
- Taylor, D. & Holmes, P. [1998] "Simple models for excitable and oscillatory neural networks," *J. Math.Biol.* 37, 419–446.
- Terman, D. [1991] "Chaotic spikes arising from a model of bursting in excitable membranes," *SIAM J. Appl.Math.* 51, 1418–1450.
- Terman, D. [1992] "The transition from bursting to continuous spiking in excitable membrane models," *J.Nonlinear Sci.* 2, 133–182.
- Terman, D. & Lee, E. [1997] "Partial synchronization in a network of neural oscillators," *SIAM J. Appl. Math.* 57, 252–293.
- Terman, D. & Wang, D. [1995] "Global competition and local cooperation in a network of neural oscillators," *Physica D81*, 148–176.
- Traub, R. D. & Miles, R. [1991] *Neuronal Networks of the Hippocampus* (Cambridge University Press, Cambridge).
- Troy, W. [1978] "The bifurcation of periodic solutions in the Hodgkin-Huxley equations," *Quart. Appl. Math.* 36, 73–83.
- Wang, X.-J. [1993] "Ionic basis for intrinsic 40 Hz neuronal oscillations," *NeuroReport* 5, 221–224.
- Wang, X.-J. [1993] "Genesis of bursting oscillations in the Hindmarsh-Rose model and homoclinicity to a chaotic saddle," *Physica D62*, 263–274.
- Wang, X.-J. [1998] "Calcium coding and adaptive temporal computation in cortical pyramidal neurons," *J.Neurophysiol.* 79, 1549–1566.
- Wang, X. J. & Rinzel, J. [1995] "Oscillatory and bursting properties of neurons," *Brain Theory and Neural Networks*, ed. Arbib, M. A. (The MIT press, Cambridge, MA).

Williams, T. L. & Sigvardt, K. A. [1995] "Spinal cord of lamprey: Generation of locomotor patterns," *Brain Theory and Neural Networks*, ed. Arbib, M.A. (The MIT press, Cambridge, MA).

Wilson, C. J. [1993] "The generation of natural firing patterns in neostriatal neurons," *Progress in Brain Research*, eds. Arbuthnott, G. W. & Emson, P. C. 99, pp. 277-297.

Wilson, C. J. & Kawaguchi, Y. [1996] "The origins of two-state spontaneous membrane potential fluctuations of neostriatal spiny neurons," *J. Neurosci.* 16,

2397-2410.

Wilson, H. R. & Cowan, J. D. [1972] "Excitatory and inhibitory interaction in localized populations of model neurons," *Biophys J.* 12, 1-24.

Wilson, M. A. & Bower, J. M. [1989] "The simulation of large scale neural networks," *Methods in Neuronal Modeling*, eds. Koch, C. & Segev, I. (The MIT Press, Cambridge, MA).

Wu, H.-Y. & Baer, S. M. [1998] "Analysis of an excitable dendritic spine with an activity-dependent stem conductance," *J. Math. Biol.* 36, 569-592.